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The Dynamics of Firm Behavior Under Alternative Cost Structures

By GEORGE A. HAY*

A large and growing number of studies attempt to determine the important factors affecting firms' decisions with respect to price, output, and inventories. A striking feature of this literature is the embarrassingly large number of alternative models—all allegedly consistent with the principles of profit maximization—which are used to justify various reduced form or behavioral equations to be estimated with the appropriate firm or industry data.

It is rare, however, that the equations to be estimated are derived rigorously from the underlying model. Because of this, the restrictions placed on the equations to be estimated are often limited at worst to nothing more than specifying which variables should be included in the regression, and at best to fixing the algebraic signs of some of the coefficients. As a consequence, it is frequently difficult or impossible to discriminate among different models involving the same list of variables.

The present paper is concerned with these problems. In it, I derive the profit-maximizing decision rules implied by a variety of alternative cost structures. The goal of the study is to demonstrate that different assumptions about the cost structure, perhaps equally plausible on a priori grounds, imply differences in the corresponding optimizing behavior, and to point out specifically what those differences are. In addition, since differences among the decision rules are interesting not only in themselves (for purposes of regression analysis) but also because of the differences they imply in the response of firms to changes in the environ-

ment, the decision rules corresponding to the various cost structures are subjected to a dynamic analysis in which various patterns of demand are simulated, and the different behavior patterns compared.

The ultimate goal of the study is to lead to models of the firm that have superior explanatory and predictive power to those encountered to date. Even if that promise is not fulfilled, however, the study should lead to an increased understanding of the way various elements of the objective function interact to generate optimal decisions for the important variables over which the firm has control.

I

In an early and pathbreaking attempt to determine optimal behavior for a firm with a relatively complex cost structure, Charles Holt et al. employed the z -transform approach to generate cost minimizing linear decision rules for production, finished goods inventories, and the size of the work force, for a manufacturing firm whose cost structure could be approximated by a quadratic function. Their work was extended by Gerald Childs who stressed the separate treatment of inventories and unfilled orders. In my earlier article in this *Review*, I included price as a decision variable, thus converting the problem into one of profit maximization. In addition both Childs and I used the results of the model to derive regression equations for use with the appropriate data.

Within the general context of the Holt-Childs-Hay model, there are a number of alternative specifications possible. Although each of the authors justifies the particular specification on a priori grounds, the case is not so strong as to rule out the possibility that an alternative specification might possess improved explanatory and predictive

* Assistant professor of economics, Yale University. I am grateful to Steven Slutsky for research and programming assistance and to Howard Kunreuther and a referee for helpful comments. An earlier version of this paper was presented at The Second World Econometric Congress, Cambridge, England, September 12, 1970.

power. In any event a systematic treatment of the decision rules implied by alternative cost structures may be useful.¹ Moreover, many of the qualitative results should be applicable not only to models within this rather narrow mold but should extend in some degree to other models which deal with approximately the same list of decision variables.

This paper considers four alternative cost structures within the general class of the Holt-Childs-Hay model. I derive the decision rules implied by the various specifications, and compare the dynamic behavior generated in response to changes in the level of demand.² The pattern of demand that is given primary attention is one in which the demand curve is assumed to shift upward by 10 percent for a single period and then return to its normal level. Different results will be obtained depending on the extent to which the change is anticipated.

A second pattern which was tried but not reported is one in which the upward shift is permanent. This experiment serves as a check on the optimality of the decision rules, since the correct equilibrium values for the decision variables are easily calculated, and can be compared with the results of the simulation.

II. Basic Model

The basic model will be presented here without extensive supporting arguments.³ The decision variables for the firm are assumed to be the rates of production and shipments, the levels of finished goods inventory and unfilled orders, and price. The following symbols will be employed:

X_t ≡ rate of production in period t

P_t ≡ price in period t

U_t ≡ level of unfilled orders (backlog) at the end of period t

U_t^* ≡ desired level of unfilled orders at the end of period t

H_t ≡ level of finished goods inventories at the end of period t

H_t^* ≡ desired level of finished goods inventories at the end of period t

O_t ≡ new orders in period t (quantity demanded)

S_t ≡ shipments in period t

V_t ≡ direct unit input costs (labor, raw materials, capital rental) in period t

The following elements make up the objective function:

i) It is assumed that there exists a desired level of unfilled orders for the period t which is proportional⁴ to production during the period. Furthermore it is assumed that the firm incurs a cost for deviating from the desired level which can be expressed as a quadratic. Thus:

$$U_t^* = c_{14}X_t$$

$$C_1 = c_1(U_t - c_{14}X_t)^2$$

ii) Similarly it is assumed that there exists a desired level of finished goods inventory for period t which is proportional to shipments during the period, and a quadratic cost for deviating from the desired level. Thus:

$$H_t^* = c_{24}S_t$$

$$C_2 = c_2(H_t - c_{24}S_t)^2$$

iii) It is assumed that the firm incurs a cost for changing the rate of production which can be expressed as a quadratic. Thus:

$$C_3 = c_3(X_t - X_{t-1})^2$$

iv) It is assumed that a similar cost exists for changing prices:

$$C_4 = c_4[(P_t - V_t) - (P_{t-1} - V_{t-1})]^2$$

where V_t is a measure of unit input cost.⁵

⁴ If an additive constant is included, a constant term is added to the decision rules.

⁵ This formulation warrants a word of explanation. To begin with, ignore the V_t terms. Then if we specify that the demand curve represents the quantity that can be sold at any price when all firms charge the same price, the C_4 term expresses the risk (which may be only subjective, i.e., as perceived by the firm) that if a firm initiates a price rise it might not be followed, and if it initiates a price cut, other firms might retaliate by

¹ A similar approach, within the context of somewhat different models, was attempted by Holt and Franco Modigliani.

² Experiments were performed with shifts in the marginal cost curve but these are not reported here.

³ For additional discussion, see Hay (1970b).

v) It is assumed that quantity demanded (new orders) is a linear function of price:

$$O_t = Q_t - bP_t$$

where b is the constant slope and Q_t the quantity intercept which is assumed to shift from period to period.

vi) The variables are constrained by the following identities:

$$O_t - S_t \equiv U_t - U_{t-1}$$

$$X_t - S_t \equiv H_t - H_{t-1}$$

The constraints can be added to the cost and revenue functions along with a factor λ to discount future profits to yield a Lagrangian expression L to be maximized:

$$\begin{aligned} L = \sum_{t=1}^N \lambda^{t-1} \{ & P_t Q_t - bP_t^2 \\ & - c_1(U_t - c_{14}X_t)^2 - c_2(H_t - c_{24}S_t)^2 \\ & - V_t X_t - c_3(X_t - X_{t-1})^2 \\ & - c_4[(P_t - V_t) - (P_{t-1} - V_{t-1})]^2 \} \\ & - \delta_t(Q_t - bP_t - S_t - U_t + U_{t-1}) \\ & - \gamma_t(X_t - S_t - H_t + H_{t-1}) \end{aligned}$$

Analytic solution of the problem using the z -transform technique⁶ would be impossible, involving the solution of an eighth degree equation for the roots of the system. However, the model can be solved numerically when particular values for the cost and revenue parameters are used.⁷ The values as-

undercutting. By including V_t we introduce the notion that a change in direct input costs acts as a signal to the industry so that all firms are expected to pass on such changes in higher (or lower) prices. Each individual firm, therefore, tries to avoid price changes which are more or less than the change in direct input costs. As it turns out, the decision rules with and without V_t are identical except that with V_t included, we add to the decision rules an infinite series of terms V_{t-1} , V_t , V_{t+1} , ..., with more or less exponentially declining coefficients thereafter.

⁶ For an extended treatment of the z -transform method, see Hay and Holt.

⁷ The comments of a referee on this issue are worth repeating. "When we deal with simple static theory, optimum conditions can be expressed in terms of the equality of marginal costs and revenues. However, when

sumed are as follows:

$$b = 2.0 \quad \lambda = .99$$

$$c_{14} = c_{24} = 1.2$$

$$c_1 = 6.5 \quad c_2 = 7.5 \quad c_3 = c_4 = 10$$

The scale of the variables can be set arbitrarily and is here chosen so that the average value of price is 100, and the average value of shipments, production and new orders (in real terms) is 100. Given these values, b is chosen so that demand elasticity is 2.0. (Recall that the demand curve specifies quantity demanded from the firm when all firms charge the same price.) The values of Q and V consistent with these specifications are 300 and 50, respectively. The values of c_{14} and c_{24} reflect the observed long-run average of the ratio of finished goods plus goods-in-process inventories to shipments and of unfilled orders to production for *U.S.* manufacturing. The values of c_1 , c_2 , c_3 , and c_4 are chosen so that in any month a 10 percent deviation of the variable in question from the desired level will lead to an increase in costs equal to 10 percent of average monthly revenue, except that c_1 and c_2 are made unequal to avoid a problem of indeterminacy in one of the specifications (Case 3 below). The value of λ reflects an assumed annual discount rate of approximately 12 percent. Several different sets of parameters were tried, and the comparisons among the various models to be discussed were not significantly affected. (For further experimentation with different parameter values, see Hay (1970b)).

The decision rules which represent the solution to the optimization problem with the above cost parameters are presented in Table 1.⁸ Only the rules for production,

cost structures are complex and dynamic, the optimal strategies may be so complex that they can not even be solved analytically. Numerical calculations may be the only feasible way to explore the properties of the theory and to deduce its properties. Such calculations can help to develop dynamic theory and clarify estimation problems."

⁸ Note that the decision rules include future values of Q . However, on the basis of work by Herbert Simon and Henri Theil (1957) it is known that in cases of

(Continued)

price and inventory are presented since the rules for unfilled orders and shipments can be derived as linear combinations of those three through the constraints. The choice of which three variables to highlight is therefore arbitrary, and in empirical work might be influenced by data availability.⁹

With regard to the decision rules underlying Table 1 we note that each of the equations is dominated to a degree by the lagged value of the dependent variable, with the coefficient of lagged inventory in the inventory equation being the largest of the three. This is notable since there is no cost-of-inventory change in the model. We also note that the coefficient of X_{t-1} in the production equation is *not* equal to the coefficient of P_{t-1} in the price equation, even though the two cost-of-change parameters were set equal. (It is however true that, *ceteris paribus*, increasing the cost-of-change parameter increases the coefficient of the lagged dependent variable in the corresponding decision rule.)

One of the important effects of the decision rules is to determine what part of a shift in the demand curve will be absorbed by increasing price, and what part by an increase in output. For a permanent increase in demand it is easily shown that higher prices will absorb half the increase and higher output the rest. The results for a temporary increase are shown in Figure 1a, which traces the response of the firm to a perfectly forecast 10 percent (30 units) increase in the quantity intercept, Q_t (i.e., a shift in the demand curve so that an additional 30 units are demanded at every price).

In the period of impact, price rises enough

quadratic criterion functions with linear constraints, substituting expected values of future Q will lead to decisions which maximize expected profits.

⁹ Since there are fewer independent cost parameters than decision rule coefficients, there are constraints involving the latter which ideally should be taken into account in any empirical estimation. This raises the two practical problems of discovering the constraints (which are generally non-linear) and using them. One approach to the first problem would be to calculate the decision rules for a great many sets of cost parameters and regress the calculated coefficients on the parameter values. This would require a large number of cases to provide adequate degrees of freedom.

TABLE 1—DECISION RULE COEFFICIENTS:
ORIGINAL SPECIFICATION

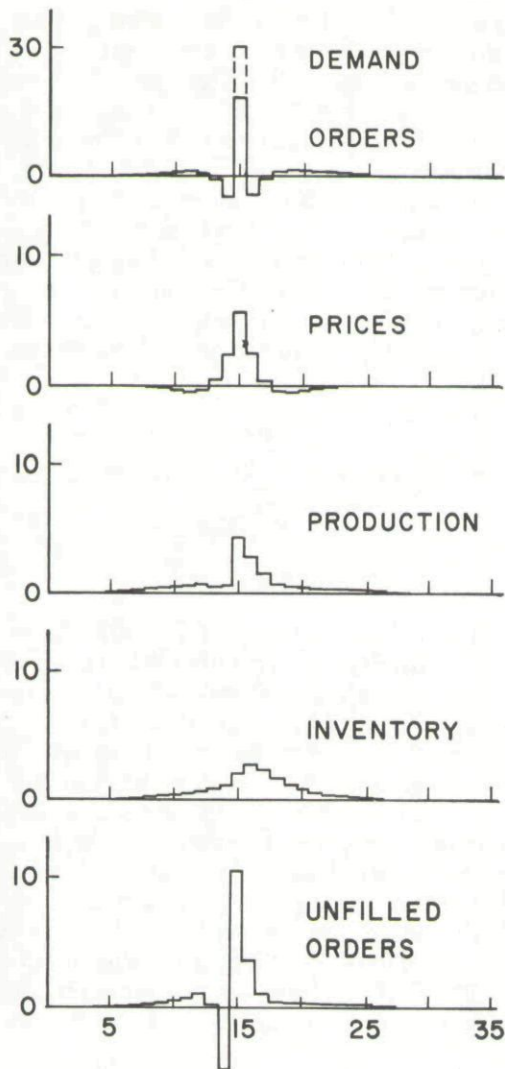
Independent Variable	Dependent Variable		
	X_t	P_t	H_t
X_{t-1}	.569	— .302	.364
P_{t-1}	— .302	.352	— .125
H_{t-1}	— .104	— .100	.504
U_{t-1}	.211	.180	.069
Q_t	.196	.198	.063
Q_{t+1}	.015	.080	.021
Q_{t+2}	— .006	.017	.007
Q_{t+3}^a	— .001	— .004	.003
V_{t-1}	.302	— .352	.125
V_t	— .389	.585	— .132
V_{t+1}	— .028	— .154	— .042
V_{t+2}	.010	— .026	— .013
V_{t+3}^a	— .002	.012	— .005

^a The coefficients of the two infinite series were calculated to period $(t+15)$ but are not all reprinted here. After one minor oscillation the coefficients decline rapidly toward zero.

to absorb slightly less than 40 percent of the increase, with the rest of the adjustment split between higher production and shipments and an increase in the backlog of unfilled orders. In previous and subsequent periods there is an additional price effect, however, so that the total amount absorbed by price turns out to be half in this case as well. Note that to assist in smoothing output, price actually falls below its long-run level when the demand increase is first anticipated (calculated here to be 15 periods in advance) and falls again after the impact has occurred. Note that while the path of price is symmetric, that of output is not, reflecting the influence of the relationship between X_t and U_t^* .

(In Figure 1b we have drawn the case for an increase in demand which is a complete surprise, i.e., not realized until the beginning of the period in which it occurs. Here price still absorbs about 40 percent in the impact period, but subsequent price cutting to smooth the transition to equilibrium output reduces the total long-run effect of price to less than one-third.)

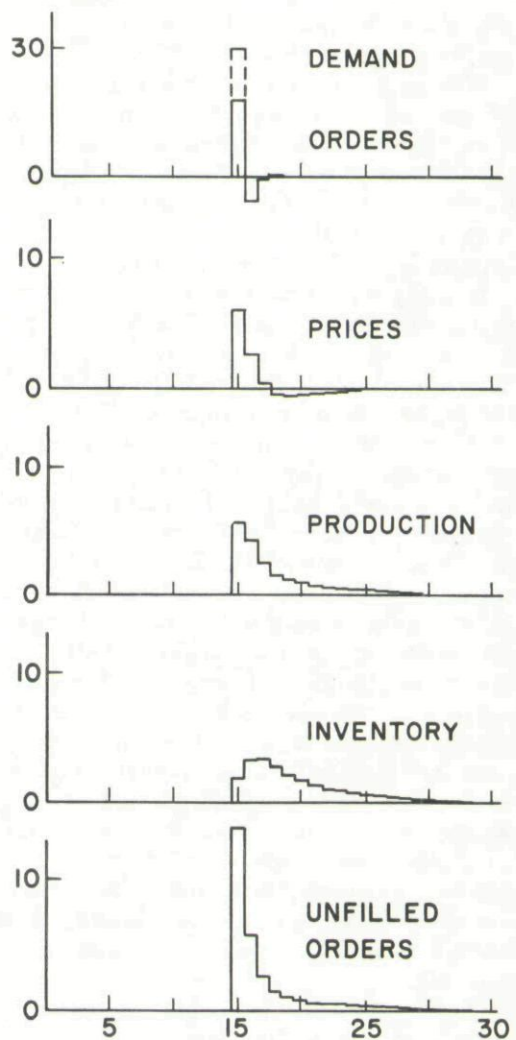
The positive coefficient of Q_t in the inventory equation suggests that a firm responds



ORIGINAL SPECIFICATION
(PERFECT FORECAST)

FIGURE 1a

to an increase in demand by adding to inventories in the period the increase occurs rather than drawing them down. Indeed, both Figures 1a and 1b confirm that observation. The source of this phenomenon is that the increased demand, because it leads to increased shipments in period t , at the same time raises the desired level of inventories.



ORIGINAL SPECIFICATION
(UNANTICIPATED DEMAND)

FIGURE 1b

The firm must compromise between shipping from inventory and deviating from its desired inventory position, or maintaining its desired inventory position and meeting the extra shipments by sharply raising output. Thus the buffer role of inventories is swamped by the attempt of the firm to maintain its desired inventory position. Some of the spe-

cifications introduced below attempt to modify that result.¹⁰

We might also note that production, price, shipments and unfilled orders all peak in the impact period, while the peak of inventories lags one period behind. (This result depends however on the parameter values assumed. A lower value of c_{24} can move the peak of inventories back to the impact period.)

In many similar models a cost of changing inventories has appeared. (See for example Paul Darling and Michael Lovell (1965).) I have argued elsewhere (see Hay (1970a) and the response by Darling and Lovell (1970)), that inventories are primarily a by-product of production. Any lag in bringing inventories to their desired level is due to costs of changing production, since costs specifically applicable to *changes* in the level of inventories are difficult to imagine. To shed more light on this question I added a cost-of-inventory change to the original model, but it turns out that there is very little change in the decision rules (a slightly higher weight on H_{t-1} in the inventory equation) and virtually none in the response to a one-period change in demand. This result is not particularly surprising since in the original model the conflicting forces operating on inventories produce relatively little movement in that variable; therefore a cost term designed to damp inventory fluctuations should have little effect.

On the other hand, in some of the cases considered below we generate significant movement of inventories and a damping force might be expected to have a more substantial impact. All of the specifications were run with a cost-of-inventory change added with the result that inventory movements became considerably damped (due to a much higher coefficient of H_{t-1} in the inventory equation) and with production absorbing much more of the adjustment burden for short period shifts in demand.

III. Alternative Specifications

As mentioned above, the appropriate and exact specification for a model of this type

¹⁰ Unfilled orders are not subject to this cancelling out since the buffer role and the effort to bring unfilled orders to their desired level both act in the same direction.

is difficult to determine on a priori grounds, and several alternative versions can be defended equally well. However, different specifications result in different decision rules and may result in substantially different behavior in response to movements in demand. It is useful therefore to trace through the implications of alternative specifications to highlight their differences. Moreover, the exercise may yield additional insight into the way in which the various parts of the model interact to generate the optimizing decision rules.

Case 2

As the first alternative we assume:

$$U_t^* = c_{14}X_{t+1}$$

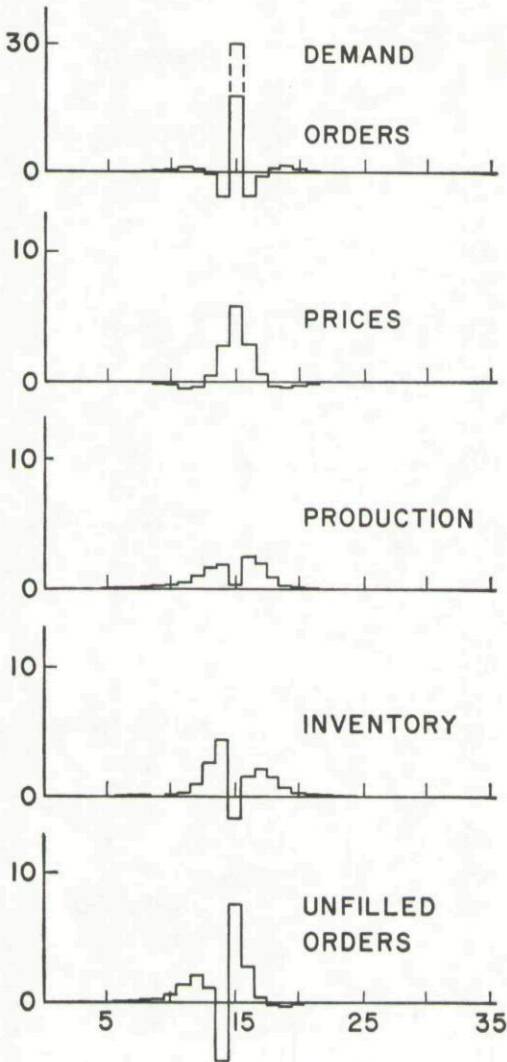
$$H_t^* = c_{24}S_{t+1}$$

The object of this specification is to test the sensitivity of the results to the lag structure. A problem with making the desired end-of-period levels proportional to activity during the period is that inventories tend to overreact to changes in demand, and their role as a buffer stock is effectively cancelled. For this and other reasons it may be interesting to introduce a lead of one period into the desired relationship. (We might note that both versions are observed in the literature, without much attention to the distinction.)

The decision rules corresponding to this specification are presented in Table 2. Sev-

TABLE 2—DECISION RULE COEFFICIENTS
 $U_t^* = c_{14}X_{t+1}$, $H_t^* = c_{24}S_{t+1}$

Independent Variable	Dependent Variable		
	X_t	P_t	H_t
X_{t-1}	.438	-.135	.415
P_{t-1}	-.135	.309	-.014
H_{t-1}	-.064	-.178	.220
U_{t-1}	.412	.092	.242
Q_t	.064	.212	-.083
Q_{t+1}	.028	.105	.091
Q_{t+2}	.012	.023	.044
Q_{t+3}	.003	-.012	.009
V_{t-1}	.135	-.309	.014
V_t	-.137	.552	.146
V_{t+1}	-.054	-.202	-.183
V_{t+2}	-.021	-.033	-.081
V_{t+3}	-.005	.030	-.001



$$U_t^* = c_{14} X_{t+1}, H_t^* = c_{24} S_{t+1}$$

FIGURE 2

eral items deserve comment. First, the production rule is not much changed except that the weight of Q_t is substantially lower, while the impact of future demand is increased somewhat. In the price rule the coefficient on lagged inventories is larger in absolute value and the weight on unfilled orders smaller. In the inventory decision rule the coefficient of lagged inventories is halved and that of Q_t

actually goes negative while the weight on future Q is increased.

These changes significantly affect the response of the system to a one period increase in demand, depicted in Figure 2. Since the buffer role of inventories is no longer in conflict with the desired level relation, inventories are built up in the period prior to the demand increase and then are drawn down sharply to absorb about 20 percent of the increase in Q . As a consequence, unfilled orders and production absorb less of the short-run burden and the buildup of production is more gradual, with the peak coming one period after the demand increase. There is actually a slight dip in the impact period reflecting the influence through the U^* relationship of the low level of unfilled orders in the previous period.

Case 3

Here we assume that the desired levels of inventories and unfilled orders are both long-run relationships. Thus the firm would not feel pressure to increase inventories in response to higher demand unless it were thought to be a permanent increase. Similarly, the only short-run upward pressure on unfilled orders is assumed to be the buffer motive. An extreme representation of these assumptions is:

$$U_t^* = \bar{U}, \quad H_t^* = \bar{H}$$

where \bar{U} and \bar{H} are treated as constants.

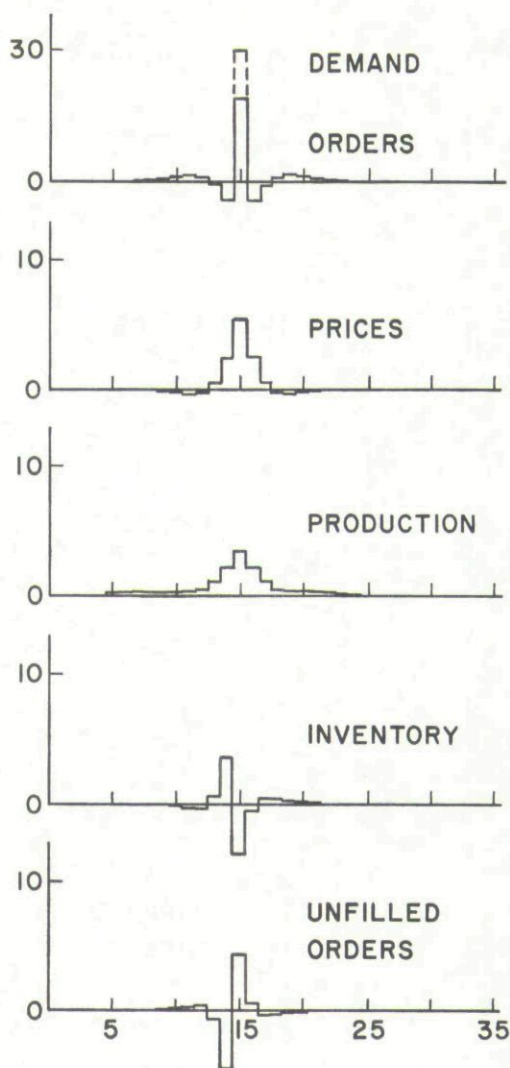
The decision rules derived from this specification are presented in Table 3 and the pattern of dynamic behavior is pictured in Figure 3. We note that the coefficients of U_{t-1} and H_{t-1} are equal and opposite in all equations so that under this particular specification it would be possible to treat orders on hand at the beginning of the period as negative inventory and lump the two into a single "net" inventory term. The optimal decisions on the current values of these variables will still be distinct, however, unless we also make $c_1 = c_2$. We also note that in the inventory decision, a positive level of unfilled orders at the beginning of the period is now a signal to lower rather than raise the level of inventories. The change results from the breakup of the chain of causation of the

original model in which a backlog of unfilled orders, by leading to more production and therefore higher shipments, caused the desired level of inventories to increase.

TABLE 3—DECISION RULE COEFFICIENTS
 $U_t^* = \bar{U}$, $H_t^* = \bar{H}$

Independent Variable	Dependent Variable		
	X_t	P_t	H_t
X_{t-1}	.698	-.256	.086
P_{t-1}	-.256	.303	.163
H_{t-1}	-.135	-.210	.207
U_{t-1}	.135	.210	-.207
Q_t	.122	.225	-.199
Q_{t+1}	.056	.092	.111
Q_{t+2}	.017	.016	.022
Q_{t+3}	.003	-.008	-.006
V_{t-1}	.257	-.303	-.163
V_t	-.253	.533	.377
V_{t+1}	-.109	-.176	-.214
V_{t+2}	-.030	-.024	-.036
V_{t+3}	-.005	.019	.015
\bar{H}	.135	.210	.793
\bar{U}	-.135	-.210	.207

We note also that two new variables, \bar{H} and \bar{U} are introduced. While the theoretical interpretation of these variables poses no problems, the issue of what to use in regression analysis is not so easily resolved. If we really mean \bar{U} and \bar{H} to be constant, they can simply be lumped into the intercept. However the specification does not require that \bar{U} and \bar{H} remain constant, but only that the decision maker regard them as being unaffected by his decisions. In the original specification, the decision maker in determining how much to produce had to adjust for the fact that in setting production he was also determining the desired level of unfilled orders, and as we have seen, this feedback effect resulted in a perverse reaction of inventories to a change in demand. For purposes of regression analysis we might still wish to use for \bar{U} and \bar{H} some measure of current or anticipated future activity, so that \bar{U} and \bar{H} might rise over time, for example, or follow a smoothed out version of the cyclical pattern of demand, while at the same time specifying the model so that the decision maker regards \bar{U} and \bar{H} as constant



$$U_t^* = \bar{U}, H_t^* = \bar{H}$$

FIGURE 3

or, at least, completely exogenous. (If the decision maker regards \bar{U} and \bar{H} as exogenous but not constant, we would need the series of the expected future values of \bar{U} and \bar{H} in the decision rules.) The variables \bar{U} and \bar{H} could also be regarded as functions of exogenous variables such as the interest rate.

The nature of the response to a temporary increase in Q depicted in Figure 3 is similar

to that of the previous case except that the movements of both inventories and unfilled orders now follow a type of symmetry about the original equilibrium. Again however, the buffer roles of those variables show up clearly.

Case 4

In the original model, the firm incurs a cost each time the production rate is changed. In particular, if the firm raises output for a single period, it incurs penalties twice—once for raising production and again for returning to the original rate. There are some costs such as hiring and firing costs and setting-up costs which are no doubt directly related to such changes. Other costs, however, such as overtime or idle time might not be adequately captured by such a specification.

As an alternative we might think in terms of a normal rate of production toward which the plant is geared, with a cost of deviating from that rate. In the extreme case, we assume that the normal rate remains constant over time so that the penalty is expressed as:

$$c_3(X_t - \bar{X})^2,$$

where \bar{X} is regarded as a constant.

The decision rules corresponding to this model are presented in Table 4. The most obvious difference is that X_{t-1} drops out as an explanatory variable and is replaced by \bar{X} . This is interesting in view of the fact that in my earlier article in this *Review* as well as in some other studies, lagged production shows surprisingly low (even negative) coefficients, especially when the models are estimated in first differences.

The time paths of production and price (Figure 4) point up clearly the effect of the change. With a temporary rise in Q , the amount of new demand absorbed by price increase is about double what it was in the original model. Second, although the total amount that has to be absorbed by increased production is correspondingly less, production peaks more sharply than in the original. This reflects the fact that with the present specification, costs can no longer be avoided

by spreading the production rise evenly over several periods.¹¹

TABLE 4—DECISION RULE COEFFICIENTS
 $C_3 = c_3(X_t - \bar{X})^2$

Independent Variable	Dependent Variable		
	X_t	P_t	H_t
X_{t-1}			
P_{t-1}	-.230	.278	-.087
H_{t-1}	-.085	-.122	.514
U_{t-1}	.223	.198	.082
Q_t	.211	.212	.077
Q_{t+1}	-.060	.110	-.027
Q_{t+2}	-.019	.035	.001
Q_{t+3}	-.002	.003	.004
V_{t-1}	.230	-.278	.087
V_t	-.429	.560	-.165
V_{t+1}	.123	-.214	.054
V_{t+2}	.037	-.062	.002
V_{t+3}	.005	-.002	-.005
\bar{X}	.688	-.433	.423

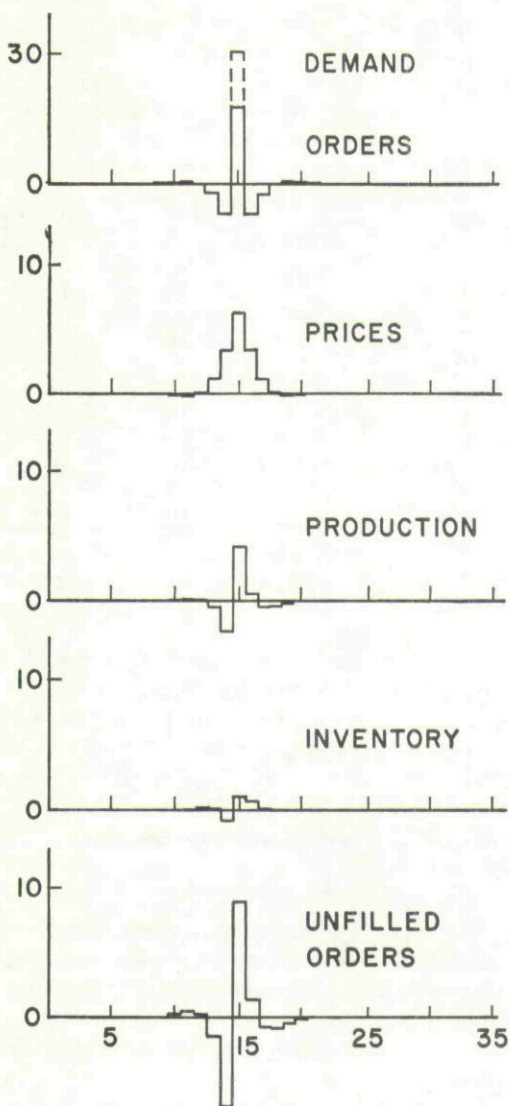
For a permanent increase in Q , price also absorbs a much larger share since production is tied to \bar{X} which by assumption does not change. Our remarks concerning \bar{U} and \bar{H} are relevant here, since it is likely that the equilibrium level of \bar{X} would tend to rise if the increase in Q were indeed permanent.

VI. Summary

Obviously a great many specifications are possible even within the narrow context of the Holt-Childs-Hay model and only a few could be presented here. Nevertheless, a number of impressions can be derived from the models tested.

In the first instance, the specification of "desired" levels of inventory and unfilled orders, while a convenience in solving the system, carries certain risks. In particular, when the desired levels are related to the levels of other decision variables in the same period, inventories and unfilled orders tend

¹¹ In the original model, the firm could absorb increased production requirements of, say, 10 units by spreading it out evenly over 10 periods and incurring costs only for the one unit change in the first and last periods. In the present model, however, a penalty must be paid in every period in which X_t differs from \bar{X} so that the disincentive to produce the required amount quickly is reduced.



$$C_3 = c_3(X_t - \bar{X})^2$$

FIGURE 4

to react too strongly to changes in those variables. This produces excessive movements of unfilled orders, where both the buffer motive and the pressure to achieve the desired level operate in the same direction. It also yields perverse movements of inventories where the two motives operate in opposite directions, with the buffer motive swamped by the attempt to maintain desired

inventories. Introducing a one-period lead into the desired relationships allows the buffer role to show up separately. A second possibility is to treat the desired levels as constant at least for moderate time intervals. A third possibility, if a stronger buffer role for inventories is desired, is to substantially reduce c_2 .

Two alternative specifications for the cost of changing production were examined and the behavior of the system was seen to be quite sensitive to the particular specification chosen. The role of price in absorbing demand is twice as important where a constant normal rate of production is specified (with penalties for producing at any other level) compared to the case where only month-to-month changes in production are costly.

A great deal of empirical work has been done in the area of firm behavior regarding prices, output, inventories, etc., and regression results have not always matched perfectly the prior hypotheses. Certainly much of the blame must lie with the data, which are at best imperfect. There is also the problem that models are built at the level of the firm while regressions are run on industry data. It is well known (see Theil (1954)) that only under extremely restrictive assumptions will the industry "decision rule" be a simple scaled-up version of the individual firms' decision rules.¹² The possibility suggested in the present paper is that a survey of the implications of alternative specifications, any one of which can probably be defended on theoretical grounds, may yield some fresh insights into previous empirical work and provide a new basis for planning future efforts.

Although the purpose of the foregoing analysis has been to explore alternative specifications in the specific context of production-inventory type models, it should be stressed that the method of analysis is relevant for a far wider range of dynamic op-

¹² However, as a referee has pointed out, if all the firms in an industry use the same production technology and face the same product and factor markets, then their decision rules should be identical and hence the conditions for perfect linear aggregation would be satisfied.

timization problems. The analysis really applies to any set of cost and constraint structures which meet the mathematical assumptions required for solution,¹³ so that its interpretation is by no means limited to production applications. One candidate would be capital theory problems although control theory has proved powerful in that context. Many problems in stabilization policy can also be made to fit the mold, as work by Theil (1964) has shown. Hopefully future work will extend the usefulness of such models even further.

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¹³ Specifically, a quadratic criterion function with linear constraints where, if future values of exogenous variables are unknown, the decision maker is content to maximize the expected value of the criterion. This method of solving dynamic optimization problems is given a general treatment in the paper by Hay and Holt.

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